FREE TRIAL

SEQUOIA
— EDUCATION —

Write your **full name** in the space above.

Mathematical Methods Examination 1

Question and Answer Book

VCE Trial Examination – Free

- Reading time is **15 minutes**
- Writing time is **1 hour**

Materials supplied

- Question and Answer Book of 12 pages
- Formula Sheet

Instructions

Students are **not** permitted to bring any technology (calculators or software) or notes of any kind into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given, unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Consider the function $f : (0, \infty) \to \mathbb{R},\ f(x) = x\log_e(x)$.

Evaluate $f'(1)$. 2 marks

b. Find $\int \frac{1}{\sqrt{1-x}}\, dx$. 2 marks

Question 2 (3 marks)

Consider the polynomial $p(x) = x^3 + 2x^2 - 3x - 6$.

a. Show that $x + 2$ is a factor of $p(x)$. 1 mark

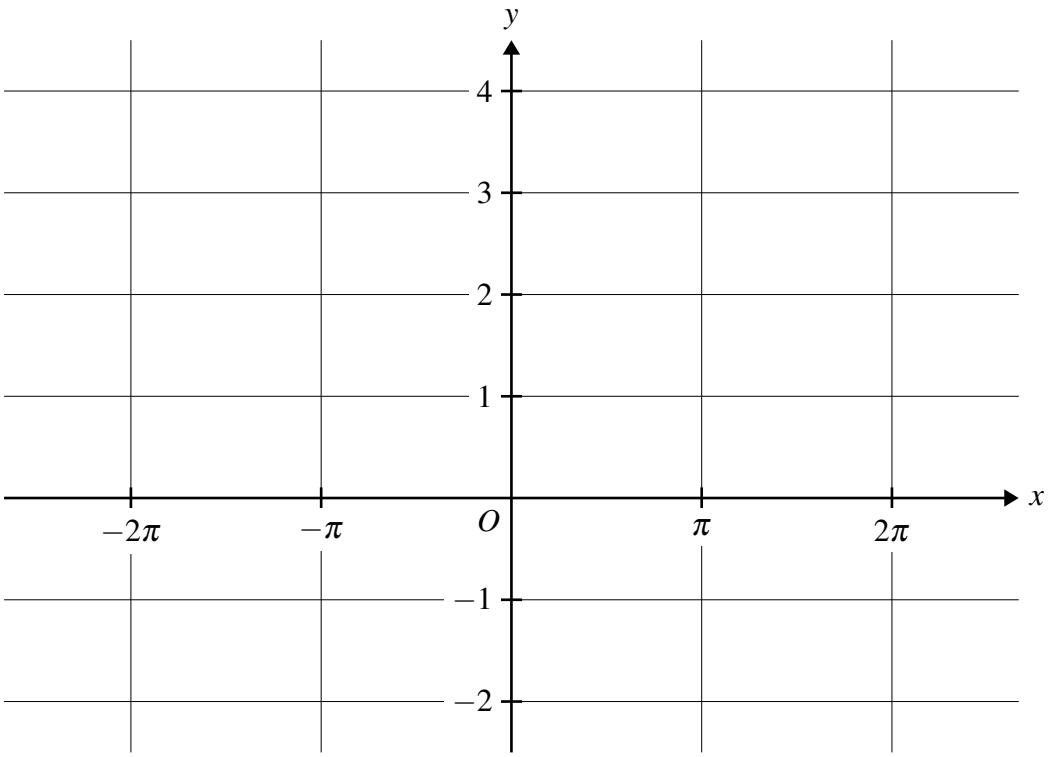
b. Solve $p(x) = 0$. 2 marks

Question 3 (5 marks)

Consider the function $f : [-2\pi, 2\pi] \to \mathbb{R},\ f(x) = 1 - 2\sin\left(\frac{x}{2}\right)$.

a. Solve $f(x) = 0$. 2 marks

b. Sketch the graph of f on the axes provided below, labelling endpoints, stationary points and axial intercepts with their coordinates. 3 marks

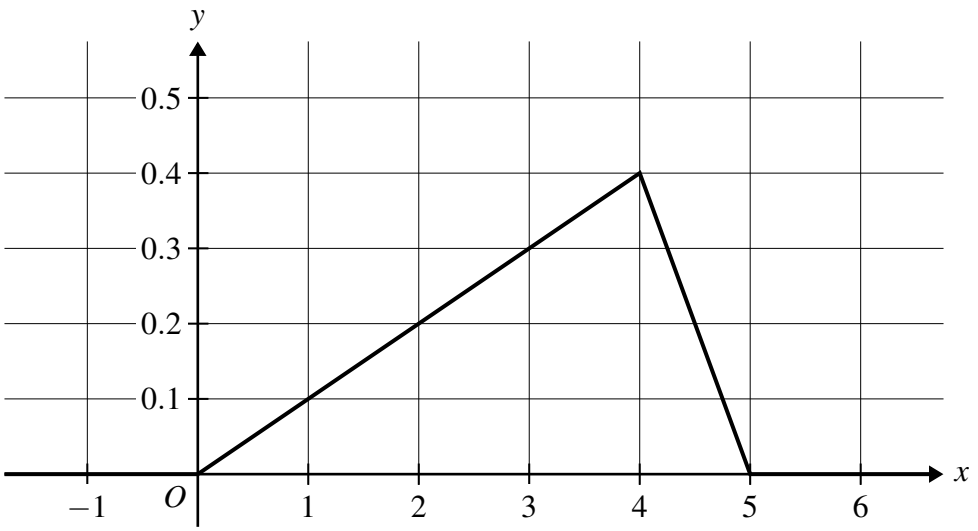


Question 4 (4 marks)

Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{x}{10}, & 0 \leq x \leq 4 \\ 2 - \frac{2x}{5}, & 4 < x \leq 5 \\ 0, & \text{otherwise} \end{cases}.$$

The graph of f is shown below.



a. Find $\Pr(1 \leq X \leq 3)$. 1 mark

b. Find the expected value of X. 3 marks

Question 5 (4 marks)

A factory uses two machines to produce batteries: Machine A and Machine B.

Machine A produces 40% of the batteries with a 5% defect rate and Machine B produces the remaining 60% with a 10% defect rate.

a. Find the probability that a randomly selected battery produced at the factory is defective. 2 marks

b. Find the probability that a randomly selected defective battery from the factory was produced by Machine B. 1 mark

Assume that the defectiveness of each battery produced by Machine B is independent.

c. Three batteries that were produced by Machine B are randomly selected.

Find the probability that exactly two of these batteries are defective. 1 mark

Question 6 (5 marks)

Consider the function $f : \mathbb{R} \setminus \{0\} \to \mathbb{R},\ f(x) = \dfrac{\cos(\pi x)}{x}$.

a. Find $f'(x)$. 1 mark

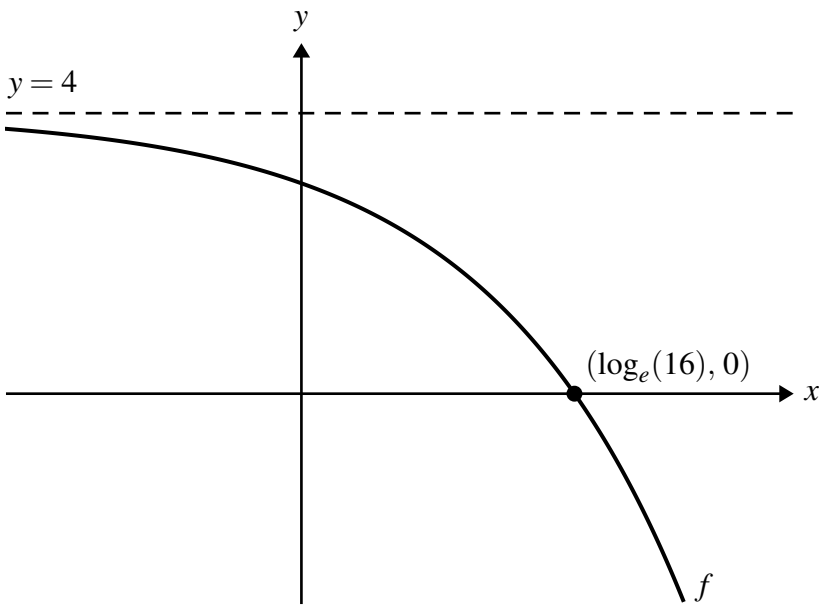
b. Find the equations of the tangents to the graph of f where $x = -1$ and $x = 1$. 2 marks

c. Find the distance between these tangents. 2 marks

Question 7 (7 marks)

Consider the function $f : \mathbb{R} \to \mathbb{R},\ f(x) = a - e^{bx}$, where a and b are positive real constants.

The graph of f has a horizontal asymptote where $y = 4$ and passes through the point $(\log_e(16), 0)$, as shown in the diagram below.

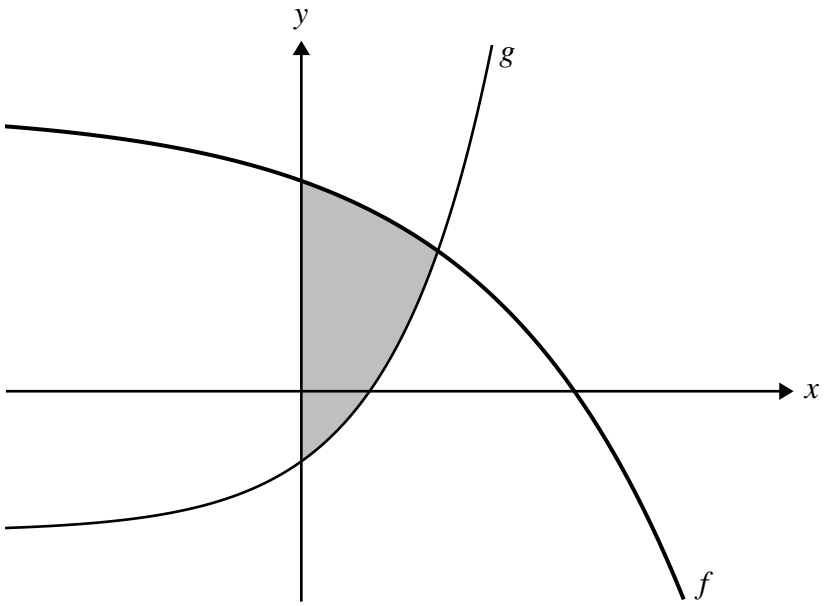


a. Show that $a = 4$ and $b = \frac{1}{2}$. 1 mark

Consider the function $g : \mathbb{R} \to \mathbb{R},\ g(x) = e^x - 2$.

b. State a sequence of transformations which maps the graph of f onto the graph of g. 2 marks

The graphs of f and g are shown below.



c. Find the area of the shaded region enclosed by the graphs of f and g, and the y-axis. 4 marks

Question 8 (8 marks)

Consider the function $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R},\ f(x) = \dfrac{x}{x+1}$.

a. Find the rule of f^{-1}, the inverse function of f. 2 marks

Consider the function $g(x) = \log_e\left(\dfrac{x}{x+1}\right)$.

b. Find the implied domain of g. 1 mark

c. Find the implied range of g. 2 marks

d. Let $p : B \to C$ and $q : A \to B$ be invertible functions.

Prove that $(p \circ q)^{-1} = q^{-1} \circ p^{-1}$. 2 marks

e. Hence or otherwise, find the rule of g^{-1}. 1 mark

End of Question and Answer Book